The Equation of State for Cold and Dense Strongly Interacting Matter *

E. S. Fraga[a,b], Y. Hatta[c,d], R. D. Pisarski[e], and J. Schaffner-Bielich[f]

[a]*Laboratoire de Physique Théorique, Université Paris XI, Bâtiment 210, 91405 Orsay Cedex, France*

[b]*Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, Rio de Janeiro, 21941-972 RJ, Brazil*

[c]*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

[d]*The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan*

[e]*Department of Physics, Brookhaven National Laboratory, Upton, NY 11973-5000, USA*

[f]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973-5000, USA*

We discuss recent results for the equation of state for cold and dense strongly interacting matter. We consider the extreme cases of very high densities, where weak-coupling approaches may in principle give reasonable results, and very low densities, where we use the framework of heavy-baryon chiral perturbation theory. We also speculate on the nature of the chiral transition and present possible astrophysical implications.

During the last decade, the investigation of strongly interacting matter under extreme conditions of temperature and density has attracted an increasing interest. In particular, the new data that started to emerge from the high-energy heavy ion collisions at RHIC-BNL, together with an impressive progress achieved by finite-temperature lattice simulations of Quantum Chromodynamics (QCD), provide some guidance and several challenges for theorists. All this drama takes place in the region of nonzero temperature and very small densities of the phase diagram of QCD (see Fig. 1). On the other hand, precise astrophysical data appear as a new channel to probe strongly interacting matter at very large densities. Compact objects, such as neutron stars, whose interior might be dense enough to accomodate deconfined quark matter, may impose strong constraints on the equation of state for QCD at high densities and low temperatures. Moreover, the first results from the lattice at nonzero values for the quark chemical potential, μ, finally start to appear (see Fig.2). Nevertheless, there is still a long way to go before they can achieve the high degree of accuracy obtained when $\mu = 0$. As they improve, they will certainly bring important contributions for a better understanding of the phase structure of QCD.

Recently, there was some progress in the study of the equation of state for QCD at zero temperature and nonzero chemical potential, especially in the regime of very high densities. In this region of the phase diagram, several different approaches, which make use of the fact that the strong coupling, α_s, is relatively weak there, seem to point in the same direction and obtain a reasonable agreement for the pressure. Here we will discuss some of these results and how they compare. Then we will jump to the opposite regime in density, namely the case of densities much smaller than the saturation density of nuclear matter, n_0, and present some interesting results provided by heavy-baryon chiral perturbation theory. Of course, the region which is most interesting for the phenomenology of compact stars lies between the two extrema. There, both techniques fail completely. However, by using results obtained within some toy models, we will speculate on some possible scenarios for the nature of the chiral transition.

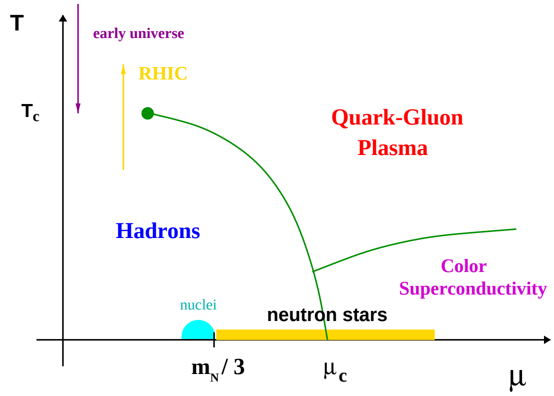

FIG. 1. Cartoon of the QCD phase diagram.

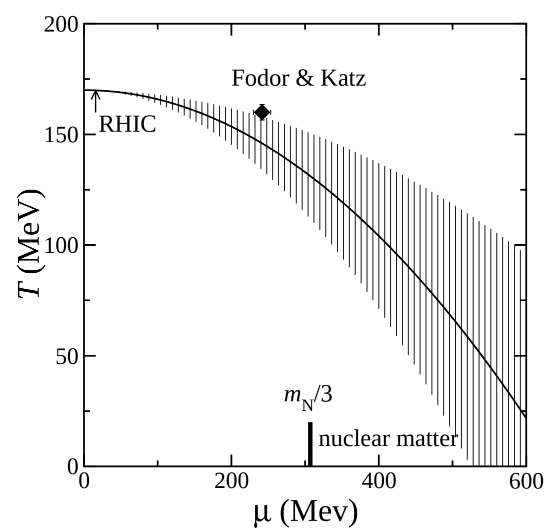

FIG. 2. Results from the lattice by Allton *et al.*, 2002.

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Let us first consider the case of cold and very dense strongly interacting matter. For high enough values of the quark chemical potential, there should be a quark phase due to asymptotic freedom. In this regime of densities, one is in principle allowed to use perturbative QCD techniques [2–4], which may be enriched by resummation methods and quasiparticle model descriptions [6–8], to evaluate the thermodynamic potential of a plasma of massless quarks and gluons. Different approaches seem to agree rasonably well for $\mu >> 1$ GeV, and point in the same direction even for $\mu \sim 1$ GeV and smaller, where we are clearly pushing perturbative QCD far below its region of applicability. This is illustrated by Figs. 3–5. However, at some point between $\mu \approx 313$ MeV, and $\mu \approx 1$ GeV, one has to match the equation of state for quark matter onto that for hadrons.

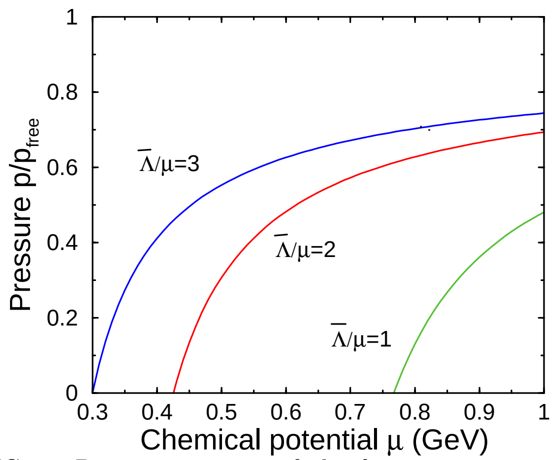

FIG. 3. Pressure in units of the free gas pressure as a function of the quark chemical potential from finite-density perturbative QCD (by Fraga, Pisarski and Schaffner-Bielich, 2001).

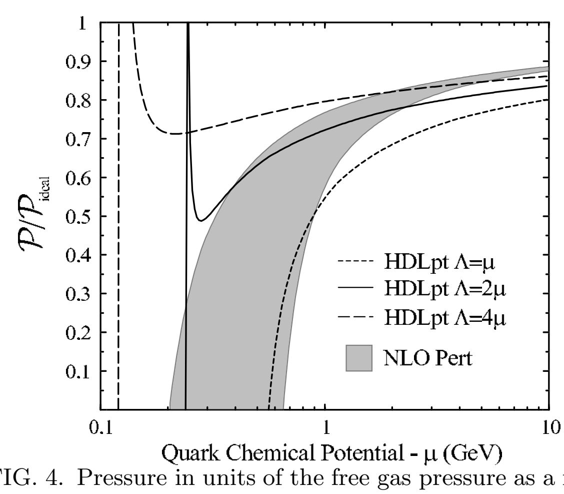

FIG. 4. Pressure in units of the free gas pressure as a function of the quark chemical potential from HDL perturbative QCD (by Andersen and Strickland, 2002).

As we argued in [4,5], depending on the nature of the chiral transition there might be important consequences for the phenomenology of compact stars. For instance, in the case of a strong first-order chiral transition (see Fig. 6), a new stable branch may appear in the mass-radius diagram for hybrid neutron stars, representing a new class of compact stars (see Fig. 7). On the other hand, for a smooth transition, or a crossover, one finds only the usual branch, generally associated with pulsar data. This is an important issue for the ongoing debate on the radius measurement of the isolated neutron star candidate RX J1856.5-3754, which might be a quark star [10]. For details, see Refs. [4] and [5].

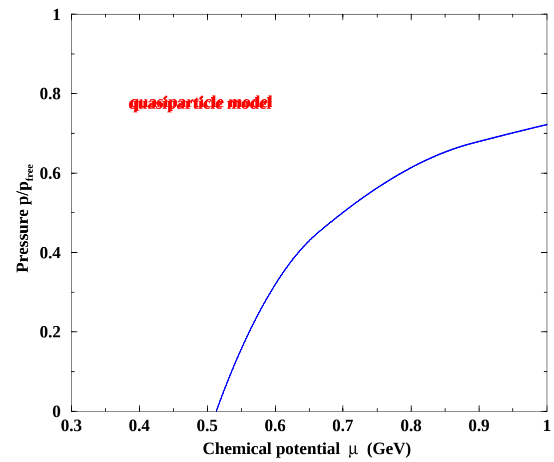

FIG. 5. Pressure in units of the free gas pressure as a function of the quark chemical potential from a quasiparticle model (by Peshier, 2002).

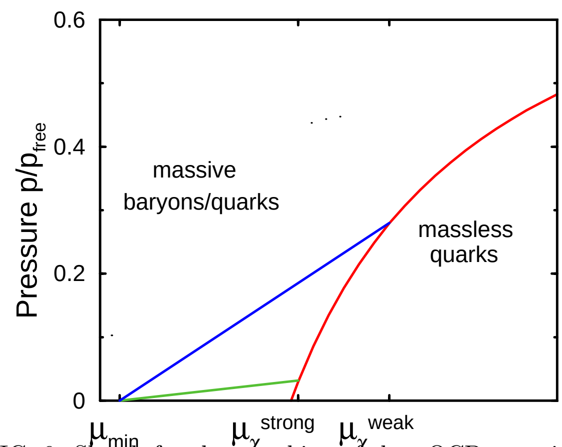

FIG. 6. Sketch for the matching of the pQCD equation of state onto that of hadrons.

For pure neutron (asymmetric) matter, which will play the role of hadrons at "low" density here, Akmal, Pandharipande and Ravenhall [11] have found that to a very good approximation we have, up to $\sim 2n_0$, the following energy per baryon:

$$\frac{E}{A} - m_N = \frac{\epsilon}{n} - m_N \approx 15 \text{ MeV} \left(\frac{n}{n_0}\right), \tag{1}$$

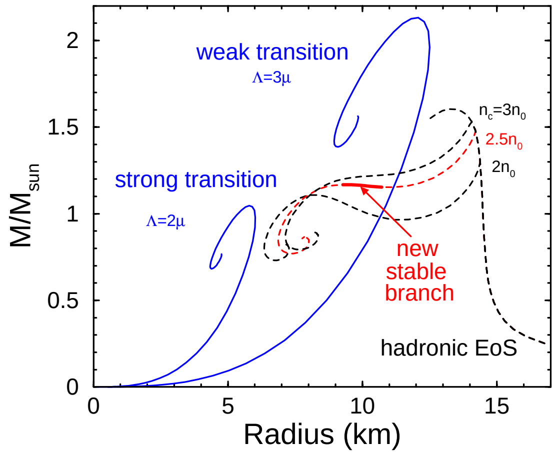

FIG. 7. Mass-radius diagram (for details, see Fraga, Pisarski and Schaffner-Bielich, 2002).

which is approximately linear in the baryon density, n. Here, $n_0 \sim 0.16$ baryons/fm^3 is the saturation density for nuclear matter. From this relation, we can extract the pressure:

$$\frac{p_{hadron}}{p_{free}} = \frac{n^2}{p_{free}} \frac{\partial}{\partial n}\left(\frac{\epsilon}{n}\right) \approx 0.04 \left(\frac{n}{n_0}\right)^2 . \qquad (2)$$

From these results, we see that: (i) even at "low" densities we have a *highly* nonideal Fermi liquid, since free fermions would give $\epsilon/n - m \sim n^{2/3}$; (ii) energies are very small on hadronic scales (if we take f_π, as a "natural scale"); (iii) energies are small not only for nuclear matter (nonzero binding energy), but even for pure neutron matter (unbound). Then, this might be a generic property of baryons interacting with pions, etc., and not due to any special tuning.

In order to investigate the pion-nucleon interaction at nonzero, but low, density, we considered the following chiral Lagrangian [12] (see also [13])

$$\mathcal{L} = \overline{\psi}_i \left[i\partial\!\!\!/ - m_N + \mu\gamma^0 - \frac{g_A}{2f_\pi}\gamma_5\gamma^\mu \vec{\tau}\cdot(\partial_\mu \vec{\pi})\right]\psi_i + \mathcal{L}_\pi^0 + \begin{bmatrix}\text{other meson}\\ \text{terms}\end{bmatrix} + \begin{bmatrix}\text{higher-order}\\ \text{terms}\end{bmatrix} , \qquad (3)$$

where $\mathcal{L}_\pi^0$ is the free Lagrangian for the pions, $\vec{\pi}$, ψ_i represent nucleons (in n_s species), and μ is the chemical potential for the nucleons. From the Goldberger-Treiman relation we have:

$$g_A = \left(\frac{f_\pi}{m_N}\right) g_{\pi NN} \qquad (4)$$

and, from the Particle Data Group, $m_N = 939\,$MeV, $m_\pi = 135\,$MeV, $f_\pi = 130\,$MeV, and $g_{\pi NN} = 13.1$ ($g_A = 1.81$)

The goal is to compute the nucleon and the pion one-loop self-energy corrections due to the medium up to lowest order in the nucleon density (only nonzero μ contributions) by using the technique of heavy-baryon chiral perturbation theory. Therefore, we adopt a non-relativistic approximation:

$$\omega_p \approx m_N + \frac{\vec{p}^{\,2}}{2m_N} + \cdots \qquad (5)$$

$$\mu \approx m_N + \frac{p_f^2}{2m_N} + \cdots \qquad (6)$$

Moreover, we assume that external legs are near the mass-shell $(p_0+\mu)^2 - \omega_p^2 \approx 0$, that pions are dilute, that we have small values of Fermi momentum, and consider only leading order in (p_f/m_N), (p_f/f_π), etc.

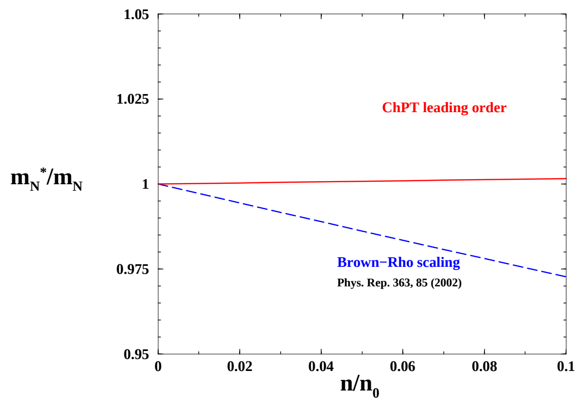

FIG. 8. Effective nucleon mass as a function of baryon density, n.

One-loop calculations within this framework provide the following result for f_π: as the Fermi momentum, p_f, increases – restoring chiral symmetry – f_π must go down. Indeed, from chiral perturbation theory, we obtain ($a \equiv (g_A^2/48\pi^2)$):

$$\frac{f_\pi(p_f)}{f_\pi} = 1 - a\,\frac{p_f^3}{m_N f_\pi^2} + \cdots = 1 - (15/939)\,(n/n_0) + \cdots \qquad (7)$$

Then, from Brown-Rho scaling one would expect that all quantities should scale in a uniform fashion. However, we obtain the following result for the nucleon mass:

$$\frac{m_N(p_f)}{m_N} = 1 + a\,\frac{p_f^3}{m_N f_\pi^2} - \frac{a}{8}\,\frac{m_\pi^2 p_f}{m_N f_\pi^2}\left\{1 + \left(\frac{m_\pi^2 + p_f^2 - p^2}{4p_f^2}\right)\log\left[\frac{m_\pi^2 + (p_f+p)^2}{m_\pi^2 + (p_f-p)^2}\right]\right\} \qquad (8)$$

Therefore, a simple and clean chiral perturbation theory calculation implies that, for very low densities, although f_π goes down as we increase the Fermi momentum, m_N goes up, violating the Brown-Rho scaling hypothesis [14]. The behavior of the effective mass as a function of density, as predicted by chiral perturbation theory and by the Brown-Rho scaling is illustrated in Fig. 8. Details of the calculations and a more complete analysis will be presented elsewhere [12].

In order to be able to make clear predictions for the phenomenology of compact stars, and to have a better understanding of this region of the QCD phase diagram, one has to find a way to describe the intermediate regime of densities in the equation of state, where perturbative calculations do not work. The study of effective field theory models might bring some insight to this problem [15].

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